ECC21 Tutorial Session on Stability and robust control of PDEs and large scale networks

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Abstract— **In this tutorial we introduce to a broad audience key concepts, results and applications of the infinite-dimensional stability theory, with a particular focus on input-to-state stability and robustness analysis. The scope of techniques which we discuss includes Lyapunov functions, nonlinear systems theory, semigroup theory, spectral decompositions and boundary control. We discuss the applications of these methods to robust stability of boundary control systems, robust control of partial differential equations and to stability of large-scale and infinite networks.**

Keywords: infinite-dimensional systems, input-to-state stability, PDE control, large-scale systems, Lyapunov methods.

I. Motivation and Outline

The concept of input-to-state stability (ISS), introduced in [23], unified the Lyapunov and input-output stability theories and revolutionized the constructive nonlinear control theory for finite-dimensional systems [11]. It played a major role in robust stabilization of nonlinear systems [2], design of robust (in terms of errors in measurements and/or quantization) nonlinear observers [14], nonlinear detectability [24], stability of nonlinear networks [7], supervisory adaptive control [3] and other fields.

The questions of robust stabilization and observation, as well as robust stability analysis of coupled systems are as important for infinite-dimensional systems as they are for finite-dimensional ones. Modern applications of control theory to chemical reactors, traffic networks, multi-body systems (e.g., robotic arms, flexible elements in aircraft), fluid-structure interactions, etc. require methods for robust stabilization of coupled systems, described by partial differential equations (PDEs). In particular, the following challenging and fascinating questions come to the foreground: How to control such complex systems, if we can physically apply controls only to the boundary of their spatial domain? How to ensure efficiency of a control design in spite of actuator and observation errors, hidden dynamics of a system and external disturbances? Under which conditions a coupled large-scale infinite-dimensional system is stable if all its components are stable?

A decade ago a development of ISS theory for infinite-dimensional systems has been started, which is going to give a systematic answer to these questions. This theory exploits a broad range of techniques and tools from such diverse fields as Lyapunov methods [15], [22], [25], semigroup and admissibility theory [5], [6], control systems [20], [21], spectral methods [8], [9], stability of networks [1], [16], etc., whose interplay resulted into efficient methods to analyze robust stability and stabilization of large-scale infinite-dimensional systems. Right now we are witnessing a fusion of the infinite-dimensional ISS theory with modern methods of PDE control, which are going to provide us with systematic methods for design of robust controllers and observers for coupled infinite-dimensional systems with heterogeneous components.

In this tutorial we present to the broad audience some of the key tools and results in this theory. We cover both functional-analytic tools coming from the operator and semigroup theory with applications to boundary control systems; PDE control, including spectral methods and pole-shifting theorems; and small-gain methods, applied to infinite nonlinear networks. We believe that this selection will represent the power and breadth of the infinite-dimensional ISS theory.

II. Intended Audience and Desired Prerequisite Knowledge

We expect the audience to include academics, postgraduate and graduate students as well as engineers interested in stability and robust control of infinite-dimensional systems.

Prerequisite knowledge of input-to-state stability is not required and all needed notions will be introduced at the tutorial. Basic knowledge in stability theory and PDEs is desired.

III. Tutorial Schedule

A general basis for this tutorial is a survey paper [18].

Individual talks are based on this survey and also on the following papers: [4], [10], [17], [12], [13], [19]

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30 min — **Stability analysis of large-scale and infinite networks.**
Andrii Mironchenko, University of Passau, Germany.

30 min — **ISS of boundary control systems.**
Andrii Mironchenko, University of Passau, Germany.

30 min — **Pole shifting theorem for parabolic systems.**
Christophe Prieur, CNRS, Univ. Grenoble Alpes, France.

30 min — **Stabilization of unstable parabolic systems via saturated controls.**
Christophe Prieur, CNRS, Univ. Grenoble Alpes, France.

IV. Speakers

The alphabetical list of instructors at this tutorial session is:

- **Andrii Mironchenko** was born in 1986. He received the M.Sc. degree in applied mathematics from the Odesa I.I. Mechnikov National University, Odesa, Ukraine, in 2008, and the Ph.D. degree in mathematics from the University of Bremen, Bremen, Germany in 2012. He has held a research position with the University of Würzburg, Würzburg, Germany and was a Postdoctoral Fellow of Japan Society for Promotion of Science (JSPS) with the Kyushu Institute of Technology, Fukuoka Prefecture, Japan (2013–2014). In 2014 he joined the Faculty of Mathematics and Computer Science, University of Passau, Passau, Germany.
  His research interests include infinite-dimensional systems, stability theory, hybrid systems and applications of control theory to biological systems and distributed control.
- **Christophe Prieur** is currently a senior researcher of the CNRS at the Gipsa-lab, Grenoble, France. He is currently a member of the EUCA-CEB, an associate editor for the AIMS Evolution Equations and Control Theory and IEEE Trans. on Control Systems Technology, a senior editor for the IEEE Control Systems Letters, and an editor for the IMA Journal of Mathematical Control and Information. He was the Program Chair of the 9th IFAC Symposium on Nonlinear Control Systems (NOLCOS 2013) and of the 14th European Control Conference (ECC 2015).
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